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Casimir repulsion and metamaterials

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Abstract

We analyze the conditions for getting the Casimir repulsion between two nonequal plates. The force between plates with magnetic permeability defined by Drude or Lorentz models is calculated. The short- and long-distance limits of the force are derived. The Casimir set-up with the hypothetical perfect matching metamaterial is discussed. We put into question the possibility of getting repulsion within the design of metamaterials based on metallic inclusions.

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1. Introduction

The study of the Casimir repulsion which always presented purely academic interest has moved in the last years to a more practical field. It is inspired by the new precision measurements of attractive Casimir force and the development of micro(nano)electromechanical machines where the Casimir repulsive force, if any, might resolve the stiction problem (see [1] and reference therein).

In the present paper, we address the Casimir repulsion between parallel plates owing to the optical properties of the material. From the Lifshitz formula [2] it follows that the force between two parallel plates is repulsive if the gap between them is filled up with a material that satisfies the inequality $\varepsilon_1(i\xi) < \varepsilon_3(i\xi) < \varepsilon_2(i\xi)$ [3], where ε_1 and ε_2 stand for the dielectric permittivities of the plates, and ε_3 corresponds to that of the filling [1, 4]. We do not consider this set-up in the present paper.

The force may become repulsive if one of the plates has a nontrivial magnetic permeability, $\mu \neq 1$. This possibility was not seriously regarded since for ‘natural’ materials, where the magnetization of the system is due to the movement of the electrons in the atoms, $\mu(\omega) = 1$ at visible range [5]. However in composite materials if the inclusions are smaller than the wavelength, but larger than the atomic size, the effective dielectric and magnetic functions can be introduced as a result of local field averaging. That is why the artificial materials [6] with magnetic response arising from micro (nano) inclusions have recently become good candidates for observing the Casimir repulsion.

The Casimir repulsion for materials with dielectric permittivity and magnetic permeability that do not depend on the frequency was considered in [7]. The case of Drude–Lorentz dispersion relations for the dielectric permittivity and magnetic permeability of the metamaterials was analyzed in [4]. The upper limits for the Casimir attractive and repulsive forces between multilayered structures at finite temperature were established. Starting from the Lifshitz formula, it was shown that at zero temperature $-7/8F_C(L) \leq F(L) \leq F_C(L)$, where F_C is the force between perfect conductors.

In [8], the Casimir repulsion due to the presence of dispersive anisotropic materials with gain was first predicted. These media are beyond the scope of our paper.

In section 2, we start from the basic formulae for the Casimir force between two nonequal plates and give the conditions for the Casimir repulsion. We consider several models for the plates and get analytic results for the force at short and long distances. In section 3, the perfect matching metamaterial is discussed. We also consider recently reported optical range metamaterial as a candidate for the Casimir repulsion. In the conclusion, we discuss the possibility of getting repulsion within the design of metamaterials based on metallic inclusions.

2. The Casimir force between nonequal mirrors

The Casimir force between two flat mirrors separated by a distance L is given by

$$F(L) = -\frac{\hbar}{4\pi c^4} \sum_{\rho} \int_0^{\infty} d\omega \omega^3 \int_1^{\infty} d\alpha \alpha^2 \frac{r_A^{\rho} r_B^{\rho}}{e^{2\alpha\omega L/c} - r_A^{\rho} r_B^{\rho}}. \quad (1)$$

Here $r^{\rho}(i\omega, \alpha)$, $\rho = \text{TE, TM}$, are the reflection coefficients at imaginary frequencies for the mirrors facing vacuum

$$r^{\text{TM}} = \frac{\sqrt{(n^2 - 1) + \alpha^2} - \varepsilon\alpha}{\sqrt{(n^2 - 1) + \alpha^2} + \varepsilon\alpha}, \quad r^{\text{TE}} = -\frac{\sqrt{(n^2 - 1) + \alpha^2} - \mu\alpha}{\sqrt{(n^2 - 1) + \alpha^2} + \mu\alpha}, \quad (2)$$

with $n = \sqrt{\varepsilon\mu}$, $\varepsilon = \varepsilon(i\omega) \geq 1$, $\mu = \mu(i\omega) \geq 1$ [5, 9]. The variable α is the ratio of the transversal wavevector at imaginary frequency $\kappa = \sqrt{\omega^2/c^2 + k^2}$ to the inverse wavelength ω/c .

The sign of the force is defined by the sign of the integrand in (1). As $|r(i\omega, \alpha)| \leq 1$, a ‘mode’ $\{\omega, \alpha\}$ gives a repulsive contribution to the force if the corresponding reflection coefficients of the mirrors A and B have opposite signs. This happens if the mirrors are different, $r_A \neq r_B$, and at least one mirror has nontrivial magnetic permeability. In [4, 10] was proved that no multilayered dielectric mirror can reverse the sign of the force.

At the lower limit of the integral over α , the reflection coefficients for TE and TM modes coincide:

$$\lim_{\alpha \rightarrow 1} r^{\text{TM}} = \lim_{\alpha \rightarrow 1} r^{\text{TE}} = (\sqrt{\mu} - \sqrt{\varepsilon})/(\sqrt{\mu} + \sqrt{\varepsilon}).$$

At the upper limit, the reflection coefficients for TE and TM modes are different:

$$\lim_{\alpha \rightarrow \infty} r^{\text{TM}} = (1 - \varepsilon)/(1 + \varepsilon) \leq 0, \quad \lim_{\alpha \rightarrow \infty} r^{\text{TE}} = -(1 - \mu)/(1 + \mu) \geq 0.$$

For fixed ω , one can find the values of α where the reflection coefficients change their signs: $\alpha_0^{\text{TM}}(\omega) = \sqrt{n^2 - 1}/\sqrt{\varepsilon^2 - 1}$, $\alpha_0^{\text{TE}}(\omega) = \sqrt{n^2 - 1}/\sqrt{\mu^2 - 1}$. We split the integral over α in (1) into two: $F_1^{\rho}(L)$ and $F_2^{\rho}(L)$. In the former the integration goes from 1 to α_0^{ρ} , and in the latter from α_0^{ρ} to ∞ .

The reflection coefficients have no extremum with respect to α . If $\mu = 1$, $\lim_{\alpha \rightarrow \infty} r^{\text{TE}} = 0$, then $r^{\text{TE}}, r^{\text{TM}} \leq 0$ at all frequencies. When $\mu < \varepsilon$, r^{TM} is always negative, falling from

$r^{\text{TM}}(\alpha = 1)$ to $r^{\text{TM}}(\alpha = \infty)$, while r^{TE} is monotonously growing from negative to positive values. If $\mu > \varepsilon$, r^{TE} remains positive and growing with α , as r^{TM} decreases from positive values at $\alpha = 1$ to negative at $\alpha = \infty$. Below we analyze the force in these situations considering the Drude or Lorentz models for the mirrors.

2.1. Two non-magnetic mirrors

When $\mu = 1$, neither r^{TM} nor r^{TE} changes the sign in the course of the integration over α from 1 to ∞ . Hence if both mirrors are non-magnetic, function (1) is always positive, and the force is attractive at all distances. As $|r^{\text{TM}}| \geq |r^{\text{TE}}|$, the contribution of the TM modes to the Casimir force (1) exceeds the TE contribution. For the mirrors described by the Lorentz model $\varepsilon_i(\omega) = 1 - \omega_{e,i}^2 / (\omega^2 - \omega_0^2 + i\gamma_{e,i}\omega)$, $i = A, B$, at short distances $L \ll \lambda_{ei}$, $\lambda_{ei} = 2\pi c / \omega_{ei}$, with $\omega_{eA} < \omega_{eB}$ we get

$$F \simeq -\frac{\hbar}{8\pi^2 L^3} \frac{\Omega_{2A}}{2} \sum_{k=0}^{\infty} G_k \left(1 - \frac{\Omega_{2A}^2}{\Omega_{2B}^2}\right)^k, \quad (3)$$

where $\Omega_{1i}^2 = \omega_{ei}^2/2$, $\Omega_{2i}^2 = \omega_{ei}^2/2 + \omega_0^2$, $i = A, B$, and $G_k = G_k(\omega_0/\omega_{eA}, \omega_0/\omega_{eB})$. The absorption in the material influences more the small frequencies which make the decisive contribution to the force at large plate separations. That is why the relaxation parameters $\gamma_{e,i}$ do not enter the short-distance asymptote.

For the mirrors described by the Drude model, $\omega_0 = 0$, the result is simplified to

$$F \simeq -\frac{\sqrt{2}}{32} \frac{\hbar \omega_{eA}}{\pi^2 L^3} \sum_{k=0}^{\infty} G_k \left(1 - \frac{\omega_{eA}^2}{\omega_{eB}^2}\right)^k, \quad (4)$$

with $G_0 \simeq 1.744$, $G_1 \simeq 0.436$, $G_2 \simeq 0.215$, $G_3 \simeq 0.133, \dots$

The long-distance limit is obtained by expanding the integrand in (1) in powers of the small parameter λ_{eA}/L or λ_{eB}/L and given by

$$F|_{L \gg \lambda_{eA}, \lambda_{eB}} = \eta F_{\text{Cas}}(L), \quad \eta \approx 1 - 4(\lambda_{eA} + \lambda_{eB}) / (3\pi L).$$

2.2. Mirror A is purely dielectric, mirror B is purely magnetic

Let the mirror A be purely dielectric, $\mu_A = 1$. Then $r_A^{\text{TE}}, r_A^{\text{TM}} \leq 0$. The mirror B is purely magnetic, $\varepsilon_B = 1$, $r_B^{\text{TE}}, r_B^{\text{TM}} \geq 0$. Then both the TE and TM modes are repulsive at all frequencies, and the force is *repulsive at all distances*.

The short-distance limit for plasma models, $\varepsilon_A = 1 + \omega_{eA}^2 / \omega^2$, $\mu_B = 1 + \omega_{mB}^2 / \omega^2$, is given by

$$F(L) \approx \frac{\sqrt{2}}{64} \frac{\hbar}{\pi c^2} \frac{(\omega_{eA}^2 \omega_{mB} + \omega_{mB}^2 \omega_{eA})}{L}. \quad (5)$$

The short-distance attraction due to the interaction between surface plasmons is absent in the present case. The TM-plasmonic mode of purely dielectric mirror and the TE-plasmonic mode of purely magnetic mirror are not coupled, and therefore do not contribute to the Casimir force. It results in the unusual short-distance asymptote (5).

At long distances, we get

$$F|_{L \gg \lambda_{eA}, \lambda_{eB}} = \eta F_{\text{Cas}}(L), \quad \eta \approx -7/8 + 7(\lambda_{eA} + \lambda_{eB}) / (6\pi L).$$

The repulsive force coinciding with the first term of this expansion was obtained by Boyer [11] for two non-dispersive mirrors with $\varepsilon_A = \infty$, $\mu_A = 1$ and $\varepsilon_B = 1$, $\mu_B = \infty$.

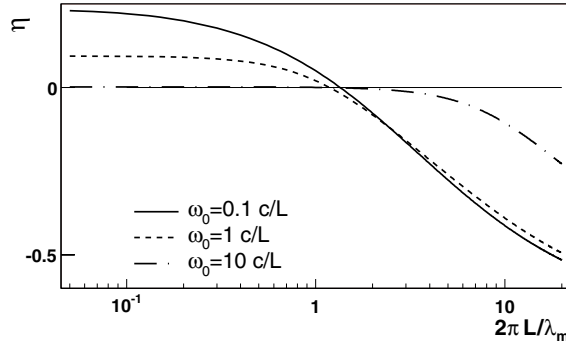


Figure 1. Reduction factor $\eta = F/F_C$ as a function of dimensionless distance $\Lambda = 2\pi L/\lambda_m = \omega_m L/c$.

2.3. Mirror A is purely dielectric, mirror B is mainly dielectric

Let the mirror A be purely dielectric, $\mu_A = 1$, with $r_A^{TE}, r_A^{TM} \leq 0$, and mirror B mainly dielectric, so that $1 \leq \mu_B \leq \varepsilon_B$ for all frequencies. Then $r_B^{TM} \leq 0$, and the contribution of the TM modes is attractive at all distances, whereas $r_B^{TE} \leq 0$ only for $\alpha < \alpha_0^{TE}$ corresponding to the mirror B. When $\alpha > \alpha_0^{TE}$, the signs of the TE reflection coefficients for the mirrors A and B are opposite. The sign of the force is the result of the balance between $F^{TM}(L) \leq 0$, $F_1^{TE}(L) \leq 0$ and $F_2^{TE}(L) > 0$.

If $\mu_B(\omega) = \varepsilon_B(\omega)$, then $\alpha_0^{TE} = 1$, and the contribution of TE modes is entirely positive, $F^{TE}(L) \geq 0$. However, as $r_B^{TM} = -r_B^{TE} \leq 0$ and $|r_A^{TE}(i\omega)| < |r_A^{TE}(i\omega)|$, the total force is attractive (negative). Consequently, when mirror B is mainly dielectric, $\mu_B(\omega) < \varepsilon_B(\omega)$, the force is attractive at all distances as well. At short distances, it is determined by the modes of TM polarization and given by (3).

2.4. Mirror A is purely dielectric, mirror B is mainly magnetic

Let the mirror A be purely dielectric, $\mu_A = 1$, with $r_A^{TE}, r_A^{TM} \leq 0$, and mirror B mainly magnetic, so that $1 \leq \varepsilon_B \leq \mu_B$ for all frequencies. Then $r_B^{TE} \geq 0$, and the contribution of the TE modes is repulsive at all distances, whereas $r_B^{TM} \geq 0$ only for $\alpha < \alpha_0^{TM}$. When $\alpha < \alpha_0^{TM}$, the signs of the TM reflection coefficients of the mirrors A and B coincide. Thus, $F^{TE}(L) \geq 0$, $F_1^{TM}(L) \geq 0$ and $F_2^{TM}(L) < 0$. The negative term $F_2^{TM}(L)$ becomes dominant for the TM modes at distances $L \leq c/(\omega_0^B \alpha_0^{TM})$. At short distances, the TM reflection coefficients are larger than the TE ones. The total force is attractive for short plate separation and repulsive at middle and long distances. For the short-distance asymptote, see equation (3).

Figure 1 gives the reduction factor of the force between purely dielectric mirror A described by the Drude model, $\varepsilon_A(i\omega) = 1 + \omega_p^2/[\omega(\gamma + \omega)]$, $\omega_p = 10c/L$, $\gamma = 0.01c/L$, $\mu_A = 1$, and mirror B with $\varepsilon_B(i\omega) = 1 + \omega_e^2/(\omega^2 + \omega_0^2 + \gamma_e \omega)$, $\mu_B(i\omega) = 1 + \omega_m^2/(\omega^2 + \omega_0^2 + \gamma_m \omega)$, where $\omega_e = \gamma_e = \gamma_m = c/L$, $\omega_0 = 0.1c/L, 1c/L, 10c/L$. Varying $\omega_m L/c$ one can see that the curves cross the horizontal axis when $\omega_m \sim \omega_e$, in other words, when the material turns into mainly magnetic. For a metamaterial with $\omega_m > \omega_e$, $\omega_0/2\pi \sim 3 \times 10^4$ GHz, the force becomes repulsive at the distances $L > 10^{-6}$ m.

3. The perfect matching metamaterial from the Casimir force viewpoint

If the real part of the refractive index, $n = n' + in''$, is negative, then the transversal wavevector, $k_z = (n^2\omega^2/c^2 - k_{\parallel}^2)^{-1/2}$, for the propagating waves $k_z(n', n'') = -k_z(-n', n'')$, and for the evanescent waves $k_z(n', n'') = k_z(-n', n'')$. It follows from the analysis of $k_z = \sqrt{\rho_1\rho_2} \exp(i\frac{\phi_1+\phi_2}{2})$ with $n' < 0$. Here,

$$\rho_1 = ([n'\omega/c - |k_{\parallel}|]^2 + [n''\omega/c]^2)^{1/2}, \quad \rho_2 = ([n'\omega/c + |k_{\parallel}|]^2 + [n''\omega/c]^2)^{1/2},$$

$$\phi_1 = \arctan \frac{n''}{-|n'| - |k_{\parallel}| \frac{c}{\omega}} + \pi, \quad \phi_2 = \arctan \frac{n''}{-|n'| + |k_{\parallel}| \frac{c}{\omega}} + \begin{cases} 0, & \text{if } n' \frac{\omega}{c} > |k_{\parallel}| \\ \pi, & \text{if } n' \frac{\omega}{c} < |k_{\parallel}| \end{cases}.$$

When the dielectric permittivity, magnetic permeability and refractive index tend to -1 [12, 13], and the absorption in the material is negligible, the ideal situation of perfect matching between material and vacuum could be achieved. It means that the transversal wavevector for the propagating waves $k_z \rightarrow -k_z^{\text{vac}}$. Consequently, the reflection coefficients of the propagating waves vanish, $r^{\text{TM}}, r^{\text{TE}} \rightarrow 0$, on the interface vacuum–metamaterial.

Let us consider two perfectly conducting mirrors, one coated with a metamaterial. Equation (1) requires that $\varepsilon(i\omega)$ and $\mu(i\omega)$ are the real positive functions in accordance with causality [5, 9]. Obviously, this condition is not satisfied for the material with a constant negative dielectric permittivity and magnetic permeability. The straightforward substitution $\mu = \varepsilon = -1$ in formula (1) for the Casimir force leads to positive force at $L > d$, and divergency at $L \leq d$. Moreover, the energy density of the electromagnetic field inside the non-dispersive material with $\varepsilon = \mu = -1$ is negative. Therefore, its very existence contradicts the Pointing theorem, $W = [\partial(\omega\varepsilon(\omega))/\partial\omega]E^2 + [\partial(\omega\mu(\omega))/\partial\omega]H^2 < 0$. On the contrary, the real parts of $\varepsilon(\omega)$ and $\mu(\omega)$ may tend to -1 at a certain frequency, leaving the energy density of the electromagnetic field positive, $W \geq 0$.

The Casimir energy for a multilayered system given in figure 2 (left) can be defined in terms of the scattering phase shift δ [14, 15]:

$$\frac{E_C}{A} = \frac{\hbar}{2} \sum_{\sigma} \int \frac{d^2k_{\parallel}}{(2\pi)^2} \left\{ \sum_{\sigma} \omega_{\sigma}^{\text{sp}}(k_{\parallel}) + \int_0^{\infty} \frac{dk_1}{\pi} \omega(k_{\parallel}, k_1) \frac{\partial\delta(k_1)}{\partial k_1} \right\}. \quad (6)$$

The first term in (6) corresponding to bound states (surface plasmons) is absent in the case of perfect mirrors.

The Maxwell equations are reduced to $\phi''(z) - \{k_{\parallel}^2 - \varepsilon_i\mu_i\omega^2/c^2\}\phi(z) = 0$, for TM modes, and $\psi''(z) - \{k_{\parallel}^2 - \varepsilon_i\mu_i\omega^2/c^2\}\psi(z) = 0$, for TE ones, $i = 1, \dots, 4$. To get the reflection and transmission coefficients and the phase shift, one has to solve the system of matching conditions, two for each interface in each polarization: $\mu_+\psi_+ = \mu_-\psi_-$, $\psi'_+ = \psi'_-$; $\varepsilon_+\phi_+ = \varepsilon_-\phi_-$, $\phi'_+ = \phi'_-$. Here, μ_{\pm} , ε_{\pm} , ϕ_{\pm} , ψ_{\pm} stand for the values of the functions when z tends to the interface from the right (left). Taking the solutions for the TE modes in the form $\phi_i = A_i e^{ik_i z} + B_i e^{-ik_i z}$, $i = 1, \dots, 4$, $B_4 = 0$, $k_i = (\varepsilon_i\mu_i\omega^2/c^2 - k_{\parallel}^2)^{1/2}$, and substituting them into the matching condition, we arrive at the system for the coefficients A_i , B_i . Then the transmission and reflection coefficients are given by the ratios $t(k_1) = A_4/A_1$, $r(k_1) = B_1/A_1$. The TM transmission and reflection coefficients are obtained by replacing $\varepsilon_i \leftrightarrow \mu_i$.

With $k_1 = k_4$, $k_2 = -k_3$, $\varepsilon_2 = \mu_2 = \mu_1 = 1$, $\varepsilon_3 = \mu_3 = n_3 = -1$, the scattering phase shift of the four-layered system is reduced to

$$2i\delta(k_1, |a'|) = \ln \frac{t(k_1)}{t(-k_1)} = \ln \frac{1 - r^2 e^{-2ik_2|a'|}}{1 - r^2 e^{2ik_2|a'|}}, \quad (7)$$

where $a' \equiv 2L_2 - L_1 - L_3 \geq 0$, $r^{\text{TM}} = (k_2\varepsilon_1 - k_1)/(k_2\varepsilon_1 + k_1)$, $r^{\text{TE}} = (k_2 - k_1)/(k_2 + k_1)$. Here the limit of infinite plate separation is subtracted. If the thickness of the metamaterial is larger

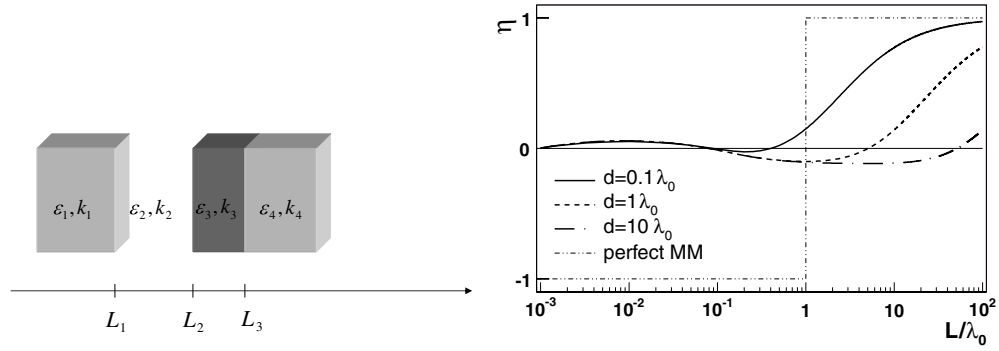


Figure 2. The reduction factor for two mirrors, one coated by metamaterial; $d = L_3 - L_2$.

than the separation of the mirrors, $a' \equiv 2L_2 - L_1 - L_3 < 0$, then $\delta(k_2, -|a'|) = -\delta(k_2, |a'|)$. When layers 1 and 3 are made of perfect metal, $r^{\text{TE}} = r^{\text{TM}} = 1$, we arrive at the Casimir result

$$E(a') = \mp A \frac{\hbar c \pi^2}{720|a'|^3} \Rightarrow F(a') = \mp A \frac{\hbar c \pi^2}{240|a'|^4},$$

with upper (lower) sign corresponding to the positive (negative) effective distance a' and the attractive (repulsive) force. The force diverges at $a' = 0$. This result cannot be recovered as a limiting case of any dispersive model of a metamaterial consistent with the Kramers–Kronig relations.

The account for the finite conductivity of the metal and dispersion in the metamaterial leads to a finite result at a plate separation equal to the thickness of the MM-coating. Figure 2 (right) gives the reduction factor for the force between two equal mirrors, one of which is coated with mainly magnetic metamaterial. The mirror A and the substrate of mirror B are described by the Drude model. The coating of mirror B is a metamaterial with effective permittivity and effective permeability given by the Drude–Lorentz model. All parameters are normalized on the position of the resonance $\omega_0 = 2\pi c/\lambda_0$, $\omega_p/\omega_0 = 10$, $\gamma/\omega_0 = \gamma_e/\omega_0 = \gamma_m/\omega_0 = 0.01$, $\omega_e/\omega_0 = 2$, $\omega_m/\omega_0 = 3$. Here, $d = L_3 - L_2$ is the thickness of the metamaterial. For comparison, we plot the reduction factor $\eta' = F(a')/F_C(a')$, $a' = (L/\lambda_0 - 1)$ for two perfect metals, one coated by a perfect matching metamaterial.

We see that if the mirror B is two-layered, the force changes the sign two times. For short and intermediate distances, $L \leq d$, the contribution of the reflections from the interface between the layer of the metamaterial and the substrate is small, and the behavior corresponding to the bulk MM-mirror is reproduced (compare with figure 1). At large distances, $L > d$, the fluctuations ‘feel’ the presence of the substrate, and the reflection coefficients approach the ones for the substrate. The force becomes attractive, achieving the values typical for metals. When the thickness of the metamaterial is smaller than its characteristic wavelength λ_0 , the layer becomes transparent for the fluctuations with $\omega > \omega_0$, and the region of repulsion is considerably narrowed (solid curve in figure 2).

Further we discuss recently reported low-loss optical metamaterial [16]. For describing the material we use the effective media approach, considering anisotropic compound material as a homogeneous media having effective dielectric and magnetic functions. To evaluate the appropriate parameters of the effective media we took the complex permittivity and permeability plots from figure 3 in [16].

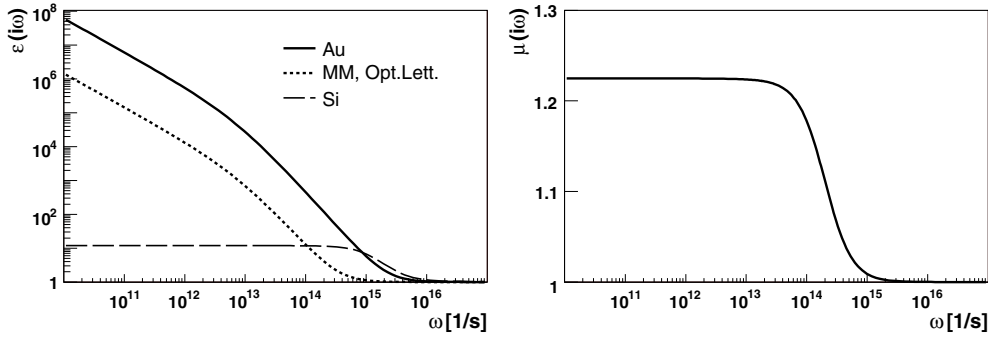


Figure 3. Left panel: $\varepsilon(i\omega)$ for metamaterial discussed in [16] in comparison with gold and silicon. Right panel: the magnetic permeability $\mu(i\omega)$ of the metamaterial; $\mu_{\text{Au}} = \mu_{\text{Si}} = 1$.

For the dielectric permittivity we have taken the Drude response with a small anti-resonance, the magnetic permeability has the Lorentz form, figure 3,

$$\varepsilon(\omega) = 1 - \frac{\omega_e^2}{\omega(\omega + i\Omega_e)} + \frac{\omega_{e1}^2}{\omega^2 - \omega_0^2 + i\Omega_{e1}\omega}, \quad \mu(\omega) = 1 - \frac{\omega_{m1}^2}{\omega^2 - \omega_0^2 + i\Omega_{m1}\omega},$$

with the position of the resonance defined by ω_0 . Though the amplitude of the magnetic resonance is larger than the corresponding dielectric one, the Drude-type term dominates in $\varepsilon(\omega)$. The magnetic permeability of the metamaterial is described by a function which is characteristic for the dielectric permittivities of the semiconductors, for example, silicon (figure 3).

The calculation of the Casimir force between golden mirror A and mirror B made of this metamaterial has been performed with the following values of the parameters: $\omega_e/2\pi = 3.6 \times 10^5$ GHz, $\Omega_e/2\pi = 8.9 \times 10^3$ GHz, $\omega_0/2\pi = 2.05 \times 10^5$ GHz, $\omega_{e1}/2\pi = 2.04 \times 10^4$ GHz, $\Omega_{e1}/2\pi = 5.03 \times 10^3$ GHz, $\omega_{m1}/2\pi = 9.72 \times 10^4$ GHz and $\Omega_{m1}/2\pi = 1.1 \times 10^4$ GHz. The metamaterial being mainly dielectric, we predict no repulsion in the present setup.

The effective medium approach is valid for the wavelengths longer than the ‘lattice constant’ of the metamaterial. To put it differently, the theoretical estimations for the force are trustable for plate separations large in comparison with the ‘lattice constant’ of the metamaterial. For more accurate results, optical data in a wide frequency range and for different incidence angles are needed.

4. Conclusion

When one of the mirrors is mainly magnetic and its magnetic permeability is described by the Drude–Lorentz model, the repulsion could be observed at the separations of the order $L \sim \lambda_0 = 2\pi c/\omega_0$. However, the force decreases rapidly with the distance. That is why to get a measurable Casimir repulsion, we look for a material with magnetic permeability $\mu \neq 1$ at optical frequencies.

Non-magnetic conductive elements, such as split ring resonators (SRR), being embedded into a dielectric lead to nontrivial magnetic response of the compound [17]. A metamaterial with effective permeability μ_{eff} is obtained when a lot of such elements are oriented in different directions and positioned as cubic lattice [6]. As the effective resonance frequency is defined by the geometric scale s of the resonator, $\omega_0 \sim 1/\sqrt{LC} \sim 1/s$ [18], the latter should be

of several hundred nanometer size. Though when the frequency of the field approaches the plasma frequency of the metal, this estimation is not valid anymore, as the electrons become insensitive to the variation of the field. Experimentally, it was shown that the geometrical scaling law for the resonance frequency breaks down at about 300 THz [19].

We seek for mainly magnetic material. The height of the magnetic resonance peak for a single resonator increases with the filling fraction. In contrast, the interaction between the inclusions makes the resonance peak broader and lower. At the same time, the metallic inclusions change the dielectric permittivity of the host media. It acquires the properties of diluted metal or highly doped dielectric with the dielectric permittivity dominated by the Drude term. In other words, providing us with the needed magnetic response, the metallic structures add as well the redundant dielectric permittivity that makes the metamaterials mainly dielectric. Consequently, they do not manifest the repulsion in the Casimir set-up. The metamaterials based entirely on dielectrics are more appropriate. That could be, for example, arrays of dielectric spheres in a dielectric matrix [20, 21] with the magnetic response arising from polaritonic resonances.

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